Q2: Evaluate $\int_0^{\pi/2} \log(\tan x + \cot x)\,dx$

Soluⁿ:

$$I = \int_0^{\pi/2} \log(\tan x + \cot x)\,dx$$

$$= \int_0^{\pi/2} \log\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) dx$$

$$= \int_0^{\pi/2} \log\left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right) dx = \int_0^{\pi/2} \log\left(\frac{1}{\sin x \cos x}\right) d$$

$$= \int_0^{\pi/2} \log\left(\left(\sin x \cos x\right)^{-1}\right) dx$$

$\log(mn)$
$= \log m + \log n$

$$= -\left(\int_0^{\pi/2} \log \sin x \, dx + \int_0^{\pi/2} \log \cos x \, dx\right)$$

$\sin 2x =$
$2 \sin x \cos x$

$$= \int_0 $$

$$= \frac{\pi}{2} \log 2 + \frac{\pi}{2} \log 2 = \pi \log 2$$

Q3: Prove that $\int_0^1 \cot^{-1}(1-x+x^2)\,dx = \frac{\pi}{2} - \log 2$

Soluⁿ:

$$I = \int_0^1 \cot^{-1}(1-x+x^2)\,dx$$

$\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$

$$= \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right) dx$$

$$= \int_0^1 \tan^{-1}\left(\frac{x-(x-1)}{1+x(x-1)}\right)$$

$$I = \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1}(x-1)\, dx$$

$\Downarrow$ $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$\text{or } I = \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1}((1-x)-1)\, dx$$

$$= \int_0^1 \tan^{-1} x\, dx + \int_0^1 \tan^{-1} x\, dx = 2\int_0^1 \underset{v}{1}\cdot \underset{u}{\tan^{-1} x}\, dx$$

Integrating by parts. $\int u \cdot v = u\int v - \int du \int v$

$$= 2\left[\tan^{-1} x \cdot x \Big|_0^1 - \int_0^1 \frac{x}{1+x^2}\, dx \right]$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log(1+x^2)\Big|_0^1\right]$$

$$\int_0^1 \frac{x\,dx}{1+x^2} = \frac{1}{2}\int\frac{dt}{t}$$

$$= \frac{1}{2}\log(t)$$

$$1+x^2 = t$$

$$2x\,dx = dt$$

$$x\,dx = \frac{dt}{2}$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right]$$

$$= \frac{\pi}{2} - \log 2 .$$ Hence proved.

Q1: $\displaystyle\int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4} + \sqrt[4]{9-x}}\,dx$

Q2: $\displaystyle\int_{-\pi/4}^{3\pi/4} \frac{\sqrt{\tan x}\,dx}{1+\sqrt{\tan x}}$